Workshop “Dispersive equations of Math Physics”
Department of Mathematics, University of Pisa
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Titles and abstracts of the talks

Paolo Antonelli
(GSSI, L'Aquila, Italy)

On the quantum Navier-Stokes system with non-trivial far-field: existence of finite energy weak solutions and low Mach number limit

Abstract: The quantum Navier-Stokes (QNS) system describes a compressible, barotropic fluid, subject to a stress tensor encoding both viscous and capillary effects. More specifically, the viscosity coefficient is proportional to the mass density and degenerates in the vacuum region, while the capillarity stress depends on the mass density and its derivatives - in a different context (quantum hydrodynamics) this is also interpreted as a correction to the fluid equations encoding quantum effects in the dynamics.

In this talk, I will review some recent results on the QNS system in the three dimensional space with non-zero conditions at infinity for the mass density.

First of all I will show the existence of finite energy weak solutions. The strategy of proof exploits the construction of a sequence of approximating solutions defined through an invading domain approach. The lack of estimates for the velocity field in the vacuum region is overcome by a suitable truncation argument which requires a careful analysis in the limit. The energy and Bresch-Desjardins entropy estimates then give the sufficient compactness. This is done is a recent preprint written in collaboration with Lars Eric Hientzsch and Stefano Spirito.

Moreover, I will also discuss the low Mach number limit for the QNS system towards the incompressible Navier-Stokes equations. The main novelty in our study is given by the analysis of the acoustic waves. Indeed the capillarity term contributes by modifying the dispersion relation. By means of a stationary phase argument we are able to provide suitable dispersive estimates showing the convergence to zero of the acoustic part, then the energy and Bresch-Desjardins entropy estimates again yield the convergence towards the incompressible dynamics. This is a joint work with Lars Eric Hientzsch and Pierangelo Marcati.

Jacopo Bellazini
(University of Sassari, Italy)

Finite energy traveling waves for the Gross-Pitaevskii equation in the subsonic regime

Abstract: We study the existence of finite energy traveling waves for the Gross-Pitaevskii equation. This problem has deserved a lot of attention in the literature, but the existence of solutions in the whole subsonic range was a standing open problem till the work of Maris s in 2013. However, such result is valid only in dimension 3 and higher. In this paper we first prove the existence of finite energy traveling waves for almost every value of the speed in the subsonic range. Our argument works identically well in dimensions 2 and 3.With this result in hand, a compactness argument could fill the range of admis-

sible speeds. We are able to do so in dimension 3, recovering the aforementioned result by Maris. The planar case turns out to be more difficult and the compactness argument works only under an additional assumption on the vortex set of the approximating solutions. Joint work with David Ruiz (Granada).

Piero D'Ancona
(Sapienza University of Rome, Italy)

Global Strichartz estimates for an inhomogeneous Maxwell system

Abstract: We study a model of electromagnetic wave propagation in a continuous medium which is asymptotically homogeneous, namely a Maxwell system with linear, isotropivc, inhomogeneous material laws. The main results are: a sharp resolvent estimate for the time harmonic case, obtained by combining Carleman and Morawetz estimates; a global smoothing estimate for the time dependent equation; and a sharp Strichartz estimate in the same range of indices as for propagation in vacuum. The result is a joint work with R.Schnaubelt (Karlsruhe)

Michele Dolce
(GSSI, L'Aquila, Italy)

Separation of time-scales in a drift-diffusion equation with radial advection on $2-d$

Abstract: Consider a passive scalar that is advected by a radial power law velocity field and it is diffused with a diffusivity coefficient $k < 1$. Physically and numerically it is known that the passive scalar is mixed on a time-scale much faster with respect to the purely diffusive one (think of milk and coffee). We identify the time-scale (sharp up to a log correction) at which mixing happens along the streamlines of the background flow. In this case, the scalar becomes firstly axisymmetric and afterward only diffusion across streamlines it is possible. The time-scale is proportional to k^{-c} where $c < 1$ depends on the strength of the background shear. The proof of the result is based on an adaptation of the hypocoercivity scheme and yield to exponential convergence towards a radially symmetric configuration. This is a joint work with Michele Coti Zelati.

Donatella Donatelli
(Department of Information Engineering, Computer Science and Mathematics, University of L'Aquila, Italy)

Dispersive behaviour in the scale limit analysis in fluid dynamics modelling

Abstract: The so called fluid dynamical models are used in different contest such as physics, astrophysics, geophysics and environmental sciences in order to model the various arising phenomena. Eliminating unwanted or unimportant modes of motion, and keeping the essential balances between flow fields, allow us to better focus and to deeper

understand the problem. Scaling and asymptotic analysis play an important role in this approach. By scaling the equations, meaning by choosing appropriately the system of the reference units, the parameters determining the behavior of the system become explicit. Asymptotic analysis provides a useful tool in the situations when certain of these parameters called characteristic numbers vanish or become infinite. On the other hand it turns out that a large part of models used in fluid dynamics rely on a formal asymptotic analysis of more complex system. Therefore it is important, from a a mathematical point of view to perform a rigorous scaling limit analysis in order to understand the behaviour of the various fluid dynamical models according to the different domains and initial conditions. The common feature of this kind of limit in the ill prepared data framework is, the presence of high frequency time oscillations along the so called acoustic waves. Those waves are supported by the gradient part of the velocity field, which, as consequence becomes infinte.

In this scenario it becomes extremely important to understand under which conditions the behaviour of this waves can be controlled and their presence is harmless. In this talk by means of different models we will show in which cases those waves have a dispersive behaviour and how dispersion may help in order to recover the necessary compactness of the velocity field.

Bibliography

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Tianxiang Gou
(Chinese Academy of Sciences, China)

Standing waves to nonlinear Chern-Simons-Schrödinger equation with prescribed L^2-norm

Abstract: In this talk, we are mainly concerned with the existence, uniqueness and dynamical behaviors of standing waves to nonlinear Chern-Simons-Schrödinger equation with prescribed L^2-norm. This is a joint work with Prof. Zhitao Zhang.

Ning-An Lai
(Lishui University, China)

Lifespan estimate for semilinear Tricomi equations

Abstract: In this talk, I will introduce some results about the lifespan estimate for semilinear generalized Tricomi equations. This is a joint work with N.M. Schiavone.

Yuan Li
(Lanzhou University, China and University of Pisa,Italy)

The Blowup solutions for the mass critical fractional Schrödinger equation
Abstract: We consider the fractional Schrödinger equation with mass critical in one dimension

$$i\psi_t = D^s\psi - |\psi|^{2s}\psi,$$

where D denotes the first order fractional derivative and $1 < s < 2$. In this paper, we prove the existence of finite-time blowup solutions with minimal mass $\|\psi_0\|_2 = \|Q\|_2$, where Q is the ground state solution of the equation $D^sQ + Q = Q^{2s}Q$. The analysis for the construction of minimal blow up elements introduced by J. Krieger, E. Lenzmann and P. Raphaël.

Sandra Lucente
(Dipartimento Interateneo di Fisica, University of Bari, Italy)

Fujita regularity exponent for scale invariant damping semilinear wave equation
Abstract: We consider the Cauchy problem

$$v_{tt} - \Delta v + \frac{\mu}{1+t}v_t + \frac{\mu}{2}\Big(\frac{\mu}{2} - 1\Big)\frac{v}{(1+t)^2} = |v|^p$$

where $v = v(t,x)$ with $x \in \mathbb{R}^n$, $\mu > 0$, $p > 1$. This equation had been studied by many authors in the last decade. Some known results describe suitable conditions on μ, p, n which implies non-existence of weak solutions or blow-up dynamic for L^p norms of the local solution. Other results establishglobal existence results provided suitable relations on μ and p. In this case Strauss and Fujita critical exponents for wave equations and heat equations comes into play. If we consider radial solution in weighted L^p spaces another parameter appears. Indeed the weight of the solution space inherits the decaying of the initial data. Let us denote by $k > 0$ the order of a polynomial decay of the initial data. Fixed μ and n, the known global existence results are valid in a particular zone of a p, k plane. In this talk, after summarizing the known results, we will describe the existence zone as a supergraph of a curve of Fujita-type. Finally we will present a new result of blowup type below such curve.

Pierangelo Marcati
(GSSI, L'Aquila, Italy)

Linear stability (and instability) for shear flows in the Compressible Euler Equations in 2-D
Abstract: We discuss the linear stability properties of the 2D isentropic compressible Euler

equations linearized around a shear flow given by a monotone profile, close to the Couette flow, with constant density, in the domain $\mathbb{T} \times \mathbb{R}$.

This is the first rigorous result in the literature regarding the compressible case. It follows results regarding the incompressible fluids by Kelvin, Rayleigh et al. at the end of the XIXth century, the analysis of Trefethen et al. in connection with pseudospectral problems.

We first investigate the Couette shear flow, showing the linear growth of the compressible part and time decay for the incompressible part (inviscid damping with slower rates).

We the extend the analysis to monotone shear flows near Couette, proving an upper bound, superlinear in time, for the compressible part of the fluid and an inviscid damping result analogous to Couette for the incompressible part.

Jmmy Alfonso Mauro
(Docente MIUR, Italy)

Estimates in Morrey-Campanato spaces of a suitable weak solution of the Navier-Stokes equations, satisfying an extra-condition

Abstract: I consider the Cauchy problem for the non-stationary Navier-Stokes equations with unit viscosity and zero body force

$$\begin{aligned} v_t - \Delta v + (v\cdot\nabla)v = -\nabla\pi \qquad & \forall\ (x,t)\ \in\ \mathbb{R}^n \times (0,T), \\ \nabla\cdot v = 0 \qquad & \forall\ (x,t)\ \in\ \mathbb{R}^n \times (0,T), \\ v(x,0) = v_0(x) \qquad & \forall\ x\ \in\ \mathbb{R}^n, \end{aligned} \tag{1}$$

with $n \geq 3$; v and π represent the unknown velocity and pressure, respectively. In my notation $(v\cdot\nabla)v = (\nabla v)v = v_k \frac{\partial}{\partial x_k} v_h$.

If $n = 3$, system (1) describes the motion of a Newtonian fluid that fills all the space $\mathbb{R}^3$.

The existence of weak solutions to the initial value problem (1) was proved by J. Leray in 1934; in particular, he introduced the first notion of weak solution for the Navier-Stokes system.

In the years 1976–77, V. Sheffer introduced the notions of *suitable* weak solution for the Navier-Stokes equations and of *generalized energy inequality*; he and others after (L. Caffarelli, R. Kohn, and L. Nirenberg) used them in developing the partial regularity theory of the Navier-Stokes system.

I consider a study of regularity of suitable weak solutions by means of the theory of Morrey-Campanato spaces. Indeed, I prove that if v is a suitable weak solution of the Cauchy problem (1) which belongs to $L^p\big(0,T;L^q(\mathbb{R}^n)\big)$ for some pair (p,q) such that

$$p,q \in [3,\infty] \quad \text{and} \quad \frac{n}{q} + \frac{2}{p} = \lambda \qquad \text{for some} \quad 1 < \lambda < \frac{n+1}{3}\,, \tag{2}$$

then, there hold two estimates, from which we can deduce

$$\begin{aligned} &\nabla v \in L^{2,k}\big(\Omega \times (\varepsilon, T)\big), \quad \text{with } k = \tfrac{n+1-3\lambda}{n+2}, \\ &v \in L^{\infty}\big(\varepsilon, T; L^{2,h}(\Omega)\big), \quad \text{with } h = \tfrac{n+1-3\lambda}{n}, \end{aligned} \tag{3}$$

where $\varepsilon \in (0, T)$, $\Omega \subset \mathbb{R}^n$ is an arbitrary bounded domain which satisfies the cone condition, $L^{2,k}\big(\Omega \times (\varepsilon, T)\big)$ and $L^{2,h}(\Omega)$ are Morrey spaces.

Raffaele Scandone
(GSSI, L'Aquila, Italy)

Global well-posedness for the non-linear Maxwell-Schrödinger system

Abstract: In this talk I will discuss the global well-posedness and the growth of Sobolev norms for the Maxwell-Schrödinger system with an additional pure-power non-linearity. The main ingredients are suitable a-priori bounds, based on a frequency-localized version of the Strichartz estimates, combined with the technique of modified energies. As an important byproduct of the a-priori bounds, it follows that the Lorentz force is well defined for ordinary solutions in the energy space.

I will also discuss the existence of global, finite energy, weak solutions to a magnetic quantum hydrodynamic system, which is formally related to the non-linear Maxwell-Schrödinger system by means of the Madelung transform.

Based on joint work with P. Antonelli and P. Marcati.

Nico Michele Schiavone
(Sapienza University of Rome, Italy)

Heat-like and wave-like behaviour of the lifespan estimates for wave equations with scale-invariant damping and mass

Abstract: In this talk we consider blow-up results and lifespan estimates in the subcritical-case for wave equations with scale-invariant damping and mass and power-nonlinearity. The focus of this talk is to show how the competition between the 'heat-like' and 'wave-like' behaviours appears not only in the definition of the critical exponent, but also in the lifespan estimates, which transit from one behaviour to the other in dependence of the damping and mass coefficients and in dependence of the exponent in the nonlinearity. Moreover, we see that the lifespan estimates can be different also under certain conditions on the initial data. This work is in collaboration with Ning-An Lai (Lishui University) and Hiroyuki Takamura (Tohoku University).

Koichi Taniguchi
(Nagoya University, Japan)

Global dynamics for semilinear heat equations in energy spaces associated with self-adjoint operators

Abstract: In this talk, we consider the global behavior of solutions to the initial-boundary value problems for semilinear heat equations with lower energy initial data than the mountain pass level in energy spaces associated with self-adjoint operators satisfying Gaussian upper bounds. This talk is based on the joint work with M. Ikeda (RIKEN / Keio University).

Mirko Tarulli
(Technical University, Sofia, Bulgaria)

Decay and Scattering in energy space for the solution of generalised Hartree equation

Abstract: We prove decay with respect some Lebesgue norms for a class of Schrödinger equations with non-local nonlinearities by showing new Morawetz inequalities and estimates. As a straightforward product we obtain large-data scattering in the energy space for the solutions to the defocusing generalized Hartree equations with mass-energy intercritical nonlinearities in any space dimensions.

Nicola Visciglia
(University of Pisa, Italy)

On the growth of solutions to a family of Schrödinger equations

Abstract: We shall discuss some growth phenomena associated with the Sobolev norms and moments of solutions to NLS.